

## FLAT SPACE MODIFIED PARTICLE DYNAMICS INDUCED BY LOOP QUANTUM GRAVITY

LUIS F. URRUTIA\*

*Departamento de Física de Altas Energías  
Instituto de Ciencias Nucleares  
Universidad Nacional Autónoma de México  
Apartado Postal 70-543  
04510 México, D.F.  
E-mail: urrutia@nuclecu.unam.mx*

Starting from an heuristic approach to the semiclassical limit in loop quantum gravity, the construction of effective Hamiltonians describing Planck length corrections to the propagation of photons and spin 1/2 fermions, leading to modified energy-momentum relations, is summarized. Assuming the existence of a privileged reference frame, we also review the determination of stringent bounds upon the parameters labelling such corrections, based upon already existing experimental data, which are found to be from five to nine orders of magnitude below the expected values.

### 1. Introduction

The possibility of probing quantum gravity effects through minute but detectable modifications to standard particle dynamics has sparked numerous investigations regarding the derivation of such corrections from a fundamental theory, together with detailed studies of the signatures identifying them.

A seminal proposal in this direction, leading to the research field now called Quantum Gravity Phenomenology, was the work of Amelino-Camelia et. al.,<sup>1</sup> suggesting that quantum gravity effects would modify the standard photon energy-momentum relation ( $c=1$ ) in the form

$$\mathbf{p}^2 = E^2 \left( 1 + \xi \left( \frac{E}{E_{QG}} \right) + O \left( \left( \frac{E}{E_{QG}} \right) \right)^2 \right), \quad (1)$$

where  $E_{QG}$  is a quantum gravity scale expected to be of the order of the Planck mass and  $\xi$  is a parameter of order one. Such modified dispersion relations have been further generalized to massive particles.

Even though the above modifications are highly suppressed by the quantum gravity scale, the possibility to amplify them by observing time-delays of high energy photons detected in gamma-ray burst originating at cosmological distances has

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been proposed.<sup>1</sup> Subsequently, additional astrophysical phenomena (like ultra high energy cosmic rays (UHECR), neutrinos, and radiation from nearby BL Lac objects, for example), together with atomic physics experiments have been studied in order to test the proposed corrections. Many of these observations were shown to impose stringent bounds upon the dimensionless parameters labelling such modifications for photons and spin 1/2 fermions.<sup>2,3,4,5</sup>

An immediate consequence of the modified dispersion relation (1) is a non-universal energy-dependent photon velocity, which implies a violation of standard Lorentz covariance. This point of view makes direct contact with a large body of research related to the experimental determination of bounds upon the parameters describing Lorentz invariance violation (LIV), which started around 1960.<sup>6,7</sup> An extension of the standard model, incorporating all possible LIV terms compatible with the known high energy interactions has been developed,<sup>8</sup> which allows to correlate the diverse experimental information. The model has recently been extended to include gravity.<sup>9</sup>

Different approaches have been followed to reproduce the proposed dynamical modifications from a fundamental theory. Among them we find those arising from loop quantum gravity (LQG),<sup>10,11,12</sup> and string theory.<sup>13,14,15</sup>

A common feature of most theories describing quantum gravity is the granular structure of space at distances of the order of the Planck length ( $\ell_P$ ), as opposed to the continuum description prevailing at large distances. In particular, one of the most striking predictions of LQG is that the eigenvalues of the area and volume operators are quantized in the corresponding units of Planck length,<sup>16</sup> thus invalidating the continuum description of space at very short distances. In analogy with wave propagation in a crystal, for example, one indeed expects that such granularity would induce dynamical corrections with respect to the propagation in the continuum.

Some points of view that have been adopted in the literature in relation to the effects of this granularity are :

(i) Dynamical corrections do arise, which signal the breaking of standard Lorentz covariance. In particular this implies the existence of a privileged reference frame (a return to the idea of the ether) which has been usually identified with the system where the cosmic microwave background radiation (CMB) looks isotropic. This point of view makes direct contact with the above mentioned experimental searches for the breaking of Lorentz invariance.

(ii) Dynamical corrections do arise, but a new relativity principle is introduced by deforming or extending the standard Lorentz transformations, so that no privileged reference frame appears. These proposals go under the name of Double Special Relativity (DSR) and basically include some maximum energy (Planck's energy for example) as an additional invariant in the modified Lorentz transformations, besides the standard speed of light as the maximum attainable velocity.<sup>17</sup>

(iii) Standard Lorentz covariance is preserved in spite of the granular character

of space at very short distances.<sup>18</sup>

In the context of LQG, the effective classical Hamiltonian for each kind of particle can be constructed as the expectation value of the corresponding quantum gravity operator in an adequate semiclassical state of the Hilbert space describing the prescribed classical matter field together with a continuum metric at large distances (flat space, for example), while retaining the quantum discreteness properties at short distances.

The first ingredient of this approach has been already developed in Thiemann's proposal for the construction of regularized versions of the required quantum Hamiltonians.<sup>19</sup> The construction of exact semiclassical states has proved to be more elusive and still constitutes an open problem, in spite of the progress already made. In fact, these more elaborate calculations provide additional support to the existence of Planck scale modifications to the dynamics.<sup>20</sup> In this way, the LQG inspired calculations of the effective Hamiltonians to be reviewed in this work are based upon some reasonable and general assumptions regarding the behavior of the required operator expectation values under the *would be* semiclassical states defined in the kinematical Hilbert space of LQG.

The paper is organized as follows. Section 2 provides a brief review of the main ideas and methods employed by Alfaro, Morales-Técotl and Urrutia to estimate such corrections in the case of photons and spin 1/2 fermions. The corresponding results are summarized in Section 3.<sup>11,12</sup> Assuming the existence of the privileged CMB frame, stringent bounds upon the LIV parameters in the fermionic case arising from clock-comparison experiments in atomic physics are reviewed in Section 4.<sup>4</sup> Finally, some closing comments are given in Section 5.

## 2. Corrections to standard particle dynamics in flat space

Central to the approach of Alfaro, Morales-Técotl and Urrutia<sup>11,12</sup> is Thiemann's regularization of the LQG Hamiltonians  $\hat{H}_\Gamma$ .<sup>19</sup> This is based upon a triangulation of space, adapted to the corresponding graphs  $\Gamma$  which define a given state in the loop representation.

The quantum Hamiltonians are defined via the holonomies and fluxes of the quantized connections and canonically conjugated momenta, respectively, around and through the faces of the tetrahedra defining the triangulation. The regularization is provided by the volume operator, with discrete eigenvalues arising only from the vertices which are common to the graph and the triangulation. Those vertices are the only ones that contribute to the action of the operator in the wave function.

Here we take an heuristic point of view, starting from the exact operator version of LQG and defining its action upon the *would be* semiclassical states through some plausible requirements.

We think of the semiclassical configuration describing a particular matter or gauge field operator  $\hat{F}$  plus gravity, as given by an ensemble of graphs  $\Gamma$ , each occurring with probability  $P(\Gamma)$ . To each of such graphs we associate a wave function

$|\Gamma, \mathcal{L}, F\rangle$  which is peaked with respect to a classical field configuration  $F$ , together with a flat gravitational metric and a zero value for the gravitational connection at large distances. In other words, the contribution of the gravitational operators inside the expectation value is estimated as

$$\begin{aligned} \langle \Gamma, \mathcal{L}, F | \dots \hat{q}_{ab} \dots | \Gamma, \mathcal{L}, F \rangle &= \delta_{ab} + O\left(\frac{\ell_P}{\mathcal{L}}\right), \\ \langle \Gamma, \mathcal{L}, F | \dots \hat{A}_{ia} \dots | \Gamma, \mathcal{L}, F \rangle &= 0 + \frac{1}{\mathcal{L}} \left(\frac{\ell_P}{\mathcal{L}}\right)^\Upsilon. \end{aligned} \quad (2)$$

The parameter  $\Upsilon \geq 0$  is a real number. Also we associate the effective Hamiltonian  $H_\Gamma = \langle \Gamma, \mathcal{L}, F | \hat{H}_\Gamma | \Gamma, \mathcal{L}, F \rangle$  to each graph. The coarse graining scale  $\mathcal{L} \gg \ell_P$  of the wave function is such that the continuous flat metric approximation is appropriate for distances much larger than  $\mathcal{L}$ , while the granular structure of space becomes relevant when probing distances smaller than  $\mathcal{L}$ . In this way, space is constructed by adding boxes of size  $\mathcal{L}^3$ , which center represents a given point  $\mathbf{x}$  in the continuum and which contain a large number of vertices of the adapted triangulation, together with the corresponding tetrahedra.

The field  $\hat{F}$ , characterized by a De Broglie wave length  $\lambda$ , is considered a slowly varying function within each box ( $\lambda > \mathcal{L}$ ) and is expanded in power series of the segments of the tetrahedra having a common vertex with the graph. The contribution of each of these segments to the expectation value is estimated by  $\ell_P$ . Also, under the expectation value, the contribution of  $\hat{F}$  is given by the value of the classical field and its derivatives at the center of the box. Gravitational variables are rapidly varying inside the box.

The total effective Hamiltonian is defined as an average over the graphs  $\Gamma$  which define the semiclassical limit:  $H = \sum_\Gamma P(\Gamma) H_\Gamma$ . This effectively amounts to average the expectation values of the gravitational variables in each box. We construct such averages in terms of the most general combinations of flat space tensors  $\delta_{ab}, \epsilon_{abc}, \dots$  which saturate the tensor structure of the classical fields together with their derivatives in each box. In this way we are imposing rotational invariance on our final effective Hamiltonian.

Next we make some general comments regarding the above procedure:

(i) our calculation has been performed in a fixed reference frame and leaves undetermined an overall numerical dimensionless coefficient in each of the calculated corrections. If these coefficients are non-zero, one would expect them to be of order one, meaning that the physics at the Planck scale has been correctly taken into account. The results can be viewed as an expansion in terms of the classical fields and their derivatives, combined with an explicitly factored out dependence upon the two scales  $\ell_P$  and  $\mathcal{L}$ .

(ii) The non-zero corrections obtained in this way have been usually interpreted as signaling a preferred reference frame together with a violation of the standard active (particle) Lorentz transformations. The advent of DSR opens up the possibility to study whether or not such modified actions can be embedded in a related frame-

work, thus recovering a modified relativity principle and eliminating the appearance of a privileged reference frame. Also, there is the possibility that a full-fledged calculation of the correction coefficients would produce a null result, thus enforcing standard Lorentz covariance.

### 3. The results

In this section we summarize the calculated effective Hamiltonians together with the corresponding modified dispersion relations, for the cases of photons and two-component spin 1/2 particles.

#### 3.1. Photons

The resulting effective Hamiltonian is<sup>11</sup>

$$\begin{aligned} H^{EM} = & \frac{1}{Q^2} \int d^3 \vec{x} \left[ \left( 1 + \theta_7 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2+2\Upsilon} \right) \frac{1}{2} \left( \underline{\vec{B}}^2 + \underline{\vec{E}}^2 \right) + \theta_2 \ell_P^2 \underline{E}^a \partial_a \partial_b \underline{E}^b + \right. \\ & + \theta_3 \ell_P^2 \left( \underline{B}^a \nabla^2 \underline{B}_a + \underline{E}^a \nabla^2 \underline{E}_a \right) + \theta_8 \ell_P \left( \underline{\vec{B}} \cdot (\nabla \times \underline{\vec{B}}) + \underline{\vec{E}} \cdot (\nabla \times \underline{\vec{E}}) \right) + \\ & \left. + \theta_4 \mathcal{L}^2 \ell_P^2 \left( \frac{\mathcal{L}}{\ell_P} \right)^{2\Upsilon} \left( \underline{\vec{B}}^2 \right)^2 + \dots \right], \end{aligned} \quad (3)$$

up to order  $\ell_P^2$ . Here  $\theta_i$  label the parameters left undetermined by our procedure and  $Q$  is the gauge (electromagnetic) coupling in Thiemann's notation.

The corresponding dispersion relation is

$$\omega_{\pm} = k \left( 1 + \theta_7 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2+2\Upsilon} - 2 \theta_3 (k \ell_P)^2 \pm 2 \theta_8 (k \ell_P) \right). \quad (4)$$

The  $\pm$  signs correspond to the two polarizations of the photon. The speed of the photon is given by  $v_{\pm}(k, \mathcal{L}) = \partial \omega_{\pm}(k, \mathcal{L}) / \partial k$ . Choosing  $\mathcal{L} = \lambda = 1/k$ , we recover the dominant helicity dependent correction found already in the seminal work of Gambini and Pullin.<sup>10</sup> As far as the  $\Upsilon$  dependent terms we have either a quadratic ( $\Upsilon = 0$ ) or a quartic ( $\Upsilon = 1$ ) correction. The only possibility to have a first order helicity independent correction amounts to set  $\Upsilon = -1/2$  which corresponds to that of Ellis et. al..<sup>14</sup> However, we do not have an interpretation for such a value of  $\Upsilon$ .

First steps towards the generalization of the Hamiltonian (3) to the Yang-Mills case have been taken.<sup>21</sup> In this work the holonomy of a non-abelian connection in an arbitrary triangular path appropriate to a face of the tetrahedra defining the triangulation has been calculated in powers of the corresponding segments of the triangulation, up to fifth order. One expects the non-abelian results to be obtained from those of the photon case just by changing ordinary derivatives  $\partial_a$  into covariant derivatives  $D_a$ . Nevertheless, this procedure does not produce a unique answer when dealing with more than one derivative, since  $[D_a, D_b] \neq 0$ . Thus, to resolve the ambiguity of such guessing one has to perform the complete calculation.

### 3.2. Two-component spin 1/2 particles

The effective Hamiltonian is<sup>12</sup>

$$H_{1/2} = \int d^3x \left[ i \pi(\vec{x}) \tau^d \partial_d \hat{A} \xi(\vec{x}) + c.c. + \frac{i}{4\hbar} \frac{1}{\mathcal{L}} \pi(\vec{x}) \hat{C} \xi(\vec{x}) + \frac{m}{2\hbar} \xi^T(\vec{x}) (i\sigma^2) (\alpha + 2\hbar\beta \tau^a \partial_a) \xi(\vec{x}) + \frac{m}{2\hbar} \pi^T(\vec{x}) (\alpha + 2\hbar\beta \tau^a \partial_a) (i\sigma^2) \pi(\vec{x}) \right], \quad (5)$$

where

$$\begin{aligned} \hat{A} &= \left( 1 + \kappa_1 \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon+1} + \kappa_2 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2\Upsilon+2} + \frac{\kappa_3}{2} \ell_P^2 \nabla^2 \right), \\ \hat{C} &= \hbar \left( \kappa_4 \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon} + \kappa_5 \left( \frac{\ell_P}{\mathcal{L}} \right)^{2\Upsilon+1} + \kappa_6 \left( \frac{\ell_P}{\mathcal{L}} \right)^{3\Upsilon+2} + \frac{\kappa_7}{2} \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon} \ell_P^2 \nabla^2 \right), \\ \alpha &= \left( 1 + \kappa_8 \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon+1} \right), \quad \beta = \frac{\kappa_9}{2\hbar} \ell_P + \frac{\kappa_{11}}{2\hbar} \ell_P \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon+1}. \end{aligned} \quad (6)$$

Here  $\kappa_i$  are the undetermined coefficients,  $\tau_i = -(i/2)\sigma^i$  (where  $\sigma^i$  are the standard Pauli matrices),  $\pi = i\xi^*$  and  $m$  is the mass of the fermion.

The corresponding dispersion relation is

$$\begin{aligned} E_{\pm}(p, \mathcal{L}) &= \left[ p + \frac{m^2}{2p} \pm \ell_P \left( \frac{1}{2} m^2 \kappa_9 \right) + \ell_P^2 \left( -\frac{1}{2} \kappa_3 p^3 + \frac{1}{8} (2\kappa_3 + \kappa_9^2) m^2 p \right) \right] \\ &\quad + \left( \frac{\ell_P}{\mathcal{L}} \right)^{\Upsilon+1} \left[ \left( \kappa_1 p - \frac{\Theta_{11} m^2}{4p} \right) \pm \ell_P \left( -\kappa_7 \frac{p^2}{4} + \Theta_{12} \frac{m^2}{16} \right) \right] \\ &\quad + \left( \frac{\ell_P}{\mathcal{L}} \right)^{2\Upsilon+2} \left( \kappa_2 p - \frac{m^2}{64p} \Theta_{22} \right), \end{aligned} \quad (7)$$

where the new coefficients  $\Theta$  are linear combinations of some  $\kappa$ 's. The velocity of propagation is  $v_{\pm}(p, \mathcal{L}) = \partial E_{\pm}(p, \mathcal{L}) / \partial p$ .

Alternative results based on a string theory inspired approach have been reported in the literature.<sup>15</sup>

### 3.3. The parameters $\mathcal{L}$ and $\Upsilon$

In order to produce numerical estimations of some of the effects arising from the previously obtained modifications to flat space dynamics, we must further fix the value of the scales  $\mathcal{L}$  and  $\Upsilon$ .

Recall that  $\mathcal{L}$  is a coarse graining scale indicating the onset distance from where the non perturbative states in the loop representation can be approximated by the classical flat metric. The propagating particle is characterized by energies which probe distances of the order of the De Broglie wave length  $\lambda$ . Just to be consistent with a description in terms of classical continuous equations it is necessary to require that  $\mathcal{L} < \lambda$ . Two distinguished cases arise: (i) the mobile scale, where we take

the marginal choice  $\mathcal{L} = \lambda$  and (ii) the universal scale, which has been introduced in the discussion of the GZK anomaly.<sup>3</sup> The consideration of the different reactions involved produces a preferred bound on  $\mathcal{L}$ :  $4.6 \times 10^{-8} \text{GeV}^{-1} \geq \mathcal{L} \geq 8.3 \times 10^{-9} \text{GeV}^{-1}$ . A recent study of the gravitational Cerenkov effect together with neutrino oscillations<sup>22</sup> yields a universal scale estimation which is consistent with the former .

Bounds for  $\Upsilon$  have been estimated based on the observation that atmospheric neutrino oscillations at average energies of the order  $10^{-2} - 10^2$  GeV are dominated by the corresponding mass differences via the oscillation length  $L_m$ . This means that additional contributions to the oscillation length, in particular the quantum gravity correction  $L_{QG}$ , should satisfy  $L_{QG} > L_m$ . This is used to set a lower bound upon  $\Upsilon$ . Within the proposed two different ways of estimating the scale  $\mathcal{L}$  of the process we obtain: (i)  $\Upsilon > 0.15$  when  $\mathcal{L}$  is considered as a mobile scale and (ii)  $\Upsilon > 1.2$  when the scale  $\mathcal{L}$  takes the universal value  $\mathcal{L} \approx 10^{-8} \text{GeV}^{-1}$ .<sup>12</sup>

#### 4. Observational bounds from spin 1/2 fermions using existing data

The previously found Hamiltonian was obtained under the assumption of flat space isotropy and was assumed to account for the fermion dynamics in a preferred reference frame, identified as the one in which the Cosmic Microwave Background looks isotropic. The earth velocity  $\mathbf{w}$  with respect to that frame has already been determined to be  $w/c \approx 1.23 \times 10^{-3}$  by COBE. Thus, in the earth reference frame one expects the appearance of signals indicating minute violations of space isotropy encoded in  $\mathbf{w}$ -dependent terms appearing in the transformed Hamiltonian or Lagrangian.<sup>4</sup> On the other hand, many high precision experimental test of rotational symmetry, using atomic and nuclear systems, have been already reported in the literature.<sup>7</sup> Amazingly enough such precision is already adequate to set very stringent bounds on some of the parameters arising from the quantum gravity corrections.

We have considered the case of non-relativistic Dirac particles and obtained corrections which involve the coupling of the spin to the CMB velocity together with a quadrupolar anisotropy of the inertial mass.<sup>4</sup> The calculation was made with the choices  $\Upsilon = 0$  and  $\mathcal{L} = 1/M$ , where  $M$  is the rest mass of the fermion. Keeping only terms linear in  $\ell_P$ , the equation of motion arising from the two-component Hamiltonian (5) can be readily extended to the Dirac case as

$$\left( i\gamma^\mu \partial_\mu + \Theta_1 m \ell_P i\gamma \cdot \nabla - \frac{K}{2} \gamma_5 \gamma^0 - m(\alpha - i\Theta_2 \ell_P \Sigma \cdot \nabla) \right) \Psi = 0, \quad (8)$$

where we have used the representation in which  $\gamma_5$  is diagonal. The spin operator is  $\Sigma^k = (i/2)\epsilon_{klm}\gamma^l\gamma^m$ ,  $K = \Theta_4 m^2 \ell_P$  and  $\alpha = 1 + \Theta_3 m \ell_P$ . The normalization has been chosen so that in the limit  $(m\ell_P) \rightarrow 0$  we recover the standard massive Dirac equation. The term  $m(1 + \Theta_3 m \ell_P)$  can be interpreted as a renormalization of the mass whose physical value is taken to be  $M = m(1 + \Theta_3 m \ell_P)$ . After this

modifications we can write an effective Lagrangian describing the time evolution as seen in the CMB frame. In order to obtain the dynamics in the laboratory frame we implement an observer Lorentz transformation in the former Lagrangian and rewrite it in a covariant looking form by introducing explicitly the CMB frame's four velocity  $W^\mu = \gamma(1, \mathbf{w}/c)$ . The result is

$$L_D = \frac{1}{2}i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - \frac{1}{2}M\bar{\Psi}\Psi + \frac{1}{2}i(\Theta_1 M\ell_P)\bar{\Psi}\gamma_\mu(g^{\mu\nu} - W^\mu W^\nu)\partial_\nu\Psi + \frac{1}{4}(\Theta_2 M\ell_P)\bar{\Psi}\epsilon_{\mu\nu\alpha\beta}W^\mu\gamma^\nu\gamma^\alpha\partial^\beta\Psi - \frac{1}{4}(\Theta_4 M\ell_P)MW_\mu\bar{\Psi}\gamma_5\gamma^\mu\Psi + h.c.. \quad (9)$$

From the work of Kostelecký and Lane<sup>23</sup> we directly obtain the non-relativistic limit of the Hamiltonian corresponding to (9), up to first order in  $\ell_P$  and up to order  $(\mathbf{w})/c^2$ , which is

$$\begin{aligned} \tilde{H} = & Mc^2(1 + \Theta_1 M\ell_P (\mathbf{w}/c)^2) + \Theta_1 M\ell_P \left[ \frac{\mathbf{w} \cdot Q_P \cdot \mathbf{w}}{Mc^2} \right] \\ & + \left( 1 + 2\Theta_1 M\ell_P \left( 1 + \frac{5}{6}(\mathbf{w}/c)^2 \right) \right) \left( \frac{p^2}{2M} + g\mu\mathbf{s} \cdot \mathbf{B} \right) \\ & + \left( \Theta_2 + \frac{1}{2}\Theta_4 \right) M\ell_P \left[ \left( 2Mc^2 - \frac{2p^2}{3M} \right) \mathbf{s} \cdot \frac{\mathbf{w}}{c} + \frac{1}{M} \mathbf{s} \cdot Q_P \cdot \frac{\mathbf{w}}{c} \right], \end{aligned} \quad (10)$$

where  $s^i = \sigma^i/2$ .

The above effective Hamiltonian has been used in the description of the valence nucleon responsible for the transitions measured in clock-comparison experiments using pairs of nuclei like  $(^{21}Ne, ^3He)$ ,<sup>24</sup> and  $(^{129}Xe, ^3He)$ ,<sup>25</sup> for example. In (10) we have not written the terms linear in the momentum since they average to zero in the nuclear bound state situation. Here  $g$  is the standard gyromagnetic factor, and  $Q_P$  is the momentum quadrupole tensor with components  $Q_{Pij} = p_i p_j - 1/3p^2\delta_{ij}$ . The terms in the second square bracket of the LHS of (10) represent a coupling of the spin to the velocity of the privileged reference frame. The first term inside the bracket has been measured with high accuracy and an upper bound for the coefficient has been found. The second term in the same bracket is a small anisotropy contribution and can be neglected. Thus we find the correction

$$\delta H_S = \left( \Theta_2 + \frac{1}{2}\Theta_4 \right) M\ell_P(2Mc^2) \left[ 1 + O\left(\frac{p^2}{2M^2c^2}\right) \right] \mathbf{s} \cdot \frac{\mathbf{w}}{c}. \quad (11)$$

The first square bracket in the LHS of (10) represents an anisotropy of the inertial mass and has been bounded in Hughes-Drever like experiments. With the approximation  $Q_P = -5/3 < p^2 > Q/R^2$  for the momentum quadrupole moment, with  $Q$  being the electric quadrupole moment and  $R$  the nuclear radius, we obtain

$$\delta H_Q = -\Theta_1 M\ell_P \frac{5}{3} \left\langle \frac{p^2}{2M} \right\rangle \left( \frac{Q}{R^2} \right) \left( \frac{w}{c} \right)^2 P_2(\cos\theta), \quad (12)$$

for the quadrupole mass perturbation, where  $\theta$  is the angle between the quantization axis and  $\mathbf{w}$ . Using  $< p^2/2M > \sim 40$  MeV for the energy of a nucleon in the last

shell of a typical heavy nucleus, together with the experimental bounds of references 24,25 we find <sup>4</sup>

$$|\Theta_2 + \frac{1}{2}\Theta_4| < 2 \times 10^{-9}, \quad |\Theta_1| < 3 \times 10^{-5}. \quad (13)$$

The second bound in (13) also imposes stringent constraints<sup>5</sup> upon some string theory inspired models that induce Planck scale corrections to field propagation.<sup>15</sup>

The above bounds on terms that were formerly expected to be of order unity, already call into question the scenarios inspired on the various approaches to quantum gravity suggesting the existence of Lorentz violating Lagrangian corrections which are linear in Planck's length. In relation to this point it is interesting to observe that a very reasonable agreement with the current AGASA ultra high energy cosmic ray (UHECR) spectrum, including the region beyond the GZK cutoff, has been recently obtained by using dispersion relations of order higher than linear in  $\ell_P$ , together with consideration of additional stringent bounds arising from the first estimations of the impact of nearby BL Lac objects and UHECR data upon LQG parameters.<sup>3</sup>

## 5. Final Comments

Since an exact construction of the semiclassical approximation in LQG is still lacking, the heuristic approach reviewed here offers a framework to make some progress towards a final understanding of the problem, together with its observational implications. On the other hand, the stringent bounds found under the assumption of the existence of a privileged frame already forbid corrections to the dynamics which are linear in  $\ell_P$ , within this scenario. From a purely phenomenological point of view one could study the possibility to alleviate these constraints by selecting an adequate parameter  $\Upsilon$ . Nevertheless a more fundamental interpretation would still be lacking. Even though the analysis of the modified dynamics in terms of a privileged reference frame has been widely used in the literature, the approach presented here is not necessarily bounded to the existence such frame. In fact, the advent of DRS has provided support to the coexistence between Planck-scale modified dynamics and an extended relativity principle. The possibility to embed our approach in a DSR-like framework needs to be further explored, having an eye on the implications that this requirement might teach us regarding the structure of the much sought semiclassical states, together with identifying new observational test that the extended covariance will demand.

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